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Using Web Pages to Teach Mathematical Modeling: Some Ideas and Suggestions

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Abstract:

Mathematical modeling can perhaps be best defined as “the process of scientific inquiry” for mathematics. This is obviously a comfortable mode for teachers of science, but is rarely seen in the mathematics classrooms of today. This paper explores the possibilities of using interactive web pages to help facilitate an understanding of practical applications based mathematics. Because the scientific process is emphasized as the general operating framework, situations where students can hypothesize and experiment, and create data tables are most valuable. Special emphasis is placed on the fact that students and teachers both need to re-conceptualize effective mathematics instruction in order to really embrace a modeling approach.

Introduction:

An important aspect of the continually changing reform movement in secondary level mathematics is that teachers are able to absorb and integrate what they have learned from both the classroom dynamics and from new research. It is perhaps most important that teachers of mathematics continue to grow with respect to the pedagogical techniques that have the greatest classroom potential. Although finding these techniques requires a great deal of effort, good teachers would certainly agree that the resources they bring to bear on behalf of their students set a foundation of success or failure for those students from both a competence standpoint and from a motivational standpoint as well.

The reform efforts of the past decade have resulted in a mass of professional documents, curriculum standards, and reports, all of which are intended to strengthen a teacher’s profile of techniques. Yet with all of the various forms of assistance, the mathematics classrooms of the 21st century will probably look very similar to those that have been so common for the past 50 years. The fact that we know so much more now than we did 50 years ago, at least from a scientific standpoint, has appeared to have very little impact on what is taught or how things are taught in the secondary math classroom. Agreed, technology has brought flavor to the mathematics classroom, but the textbooks along with their very familiar format still seem to be the preferred method of instruction. Although there are instructional perks to this classroom format, the fact that students aren’t internalizing the information would suggest that other formats merit exploration.

Instructional activities using a mathematical modeling approach have proven to be both effective and engaging for students. Additionally, some of the most valuable curriculum-application considerations in today’s mathematics classrooms can be revisited in the context of an interactive web based format that preempts the “what do we need this for” question. The mathematical modeling approach to instruction is indeed a “front heavy” technique for teachers, but allows for the kind of valuable exploration in mathematics that has been absent to date.

Classroom Considerations for Mathematical Modeling:

Mathematical modeling can perhaps be best defined as “the process of scientific inquiry” for mathematics. This is obviously a comfortable mode for teachers of science, but is rarely seen in mathematics classrooms. Students engaging in mathematical modeling activities would spend the majority of time experimenting in applied physical situations in an attempt to find patterns and consistencies in sets of data. Data sets could already exist in a number of different forms, or they could be collected as part of a classroom activity. Part of the impetus for mathematical modeling activities in the classroom is to help students understand that mathematics is not a discipline where complex solutions to problems are innately obvious or solvable in a matter of just a few minutes. In fact, any good mathematical modeling activity should be appropriately vague so that the students don't get the impression that the activity is just another textbook assignment. The teacher designing the activity has the difficult task of articulating the problem in such a way as to provide clarity without being too prescriptive. This is done to emphasize that mathematical modeling is a process of continual refinement and modification. In most cases, this process of refinement serves two distinct tasks. First, the refinements are intended to create a working model that is more efficient, faster, or more accurate in some way than any previous model. Secondly, refinement and modification are natural processes of building any axiomatic system of notation. Students in essence build their own mathematical system of notation and in turn, greater mathematical understanding. Some instructional considerations related to the use of mathematical modeling activities in the classroom are as follows:

1. Students have some control over how they approach a problem. This is not typically the case with most textbook problems.
2. Good modeling activities are adaptable to many different ability levels.
3. Good modeling activities are easily scalable to different grade levels.
4. Problem solving and mathematical modeling are different processes. Problem solving typically acts as a process oriented approach whereby students find a specific solution to a specific problem. Mathematical modeling is an experimental approach where a problem is solved and continually refined over time in order to be more efficient, faster, or more accurate. Problem solving in many cases has a solution that is either correct or incorrect. Mathematical modeling is a process where few answers are incorrect, they just require continual revision.
5. Mathematical modeling focuses primarily on the "general case." Students must at least generally understand the concept of a variable, which is why modeling activities below the fifth grade are difficult for teachers to construct.
6. Mathematical modeling activities are difficult to assess. An elegant solution may be an approach that works in a way that appears to be coincidental, but a student can justify why. Another solution to the same problem may utilize some specific procedure from the textbook, yet the student has no understanding of why they chose that method nor why it works.

The premise of the mathematical modeling concept is not that the traditional courses in the curriculum need to be replaced, but rather accented in the appropriate spots to better emphasize the practical use of the concepts we do teach. Because mathematical models can take on many forms, the processes by which problems are approached are numerous and varied. Some of the more basic modeling structures lend themselves very well to established secondary level curriculum (i.e. numerical tables and patterns, graphs, systems of equations, etc.). Others may be more algorithm-based problems that require a computer or graphing calculator as an extension. Although no one set of rules is inherent to all mathematical modeling activities, the following set of steps can act as general guidelines for students engaged in mathematical modeling activities:

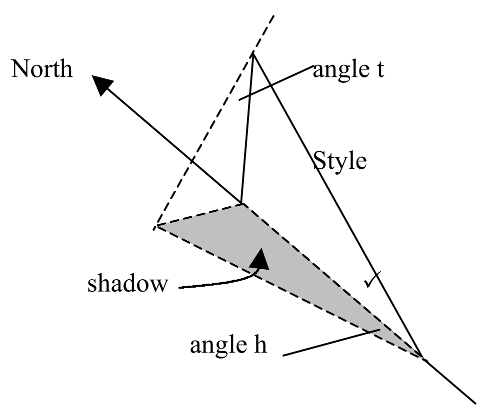
1. Identify what the problem and resulting model should look like
2. Establish the factors that affect the outcome
3. Define which of the factors are parameters and which are outcome variables
4. Establish a relationship between the parameters and the variables to derive a formula or alternately defined model or algorithm
5. Test the model with known values from previously collected data
6. Refine the model for accuracy and efficiency

Using Web Pages for Modeling Activities:

Although modeling activities in the mathematics classroom don't have to be technology driven, the interactive nature of Java applications on many web pages can provide a physical context which students can use to test conjectures and build generalizations. Because the scientific process is emphasized as the general operating framework, situations where students can hypothesize, experiment, and create data tables are most valuable. Well designed web pages using Java allow for the kind of interactive experiments needed for success without the hassle of setting up a physical lab situation. The following example illustrates a possible modeling problem that could be practically used on the web:

<u>Problem:</u> Suppose we wanted to find the time of day without using a clock. In ancient times, sundials were used for this purpose, and were fairly accurate. The first step in finding the time without using a clock is to use the relative movement of the sun and earth to predict how shadows might fall at different times of the day. Assuming that the meridian line (or noontime mark) has been established and the gnomon has been angled, we must find a way to mark the hour lines on the dial plate. Create a mathematical model that uses the angle of the sun on the style (top of the gnomon that creates the shadow) to mark hour lines on the dial plate of the sundial. Using angle ✓ as the base angle of the gnomon, and angle t as the angle of the arc the sun passes through in a given time frame, we should be able to calculate angle h by using the length of the resulting shadow. This is illustrated in figure 1.

Figure 1: Shadow used to mark the dial plate



<u>One Possible Solution:</u> Angle t is perhaps the first angle that needs to be defined. Because the earth rotates through a central angle of 360^o in 24 hours, we can assume that each hour is defined by a 15^o arc of the sun's apparent movement over the surface of the earth. This is true at any longitude. Angle t then measures 15^o for each hour away from the noon hour.

If the length of the style on the gnomon is known, the vertical side of the gnomon can be calculated as follows: Height = sin ✓(S) ; where S is the length of the style.

At 11:00, angle t = 15^o. So, if we want to mark the 11:00 hour on the dial plate, angle h can be calculated by measuring the length of the shadow from the noon mark and subsequently estimating the length of the adjacent side as equal to the length of the style, so that the tangent ratio can be used. The side of the shadow opposite angle h = (tan t)(sin ✓)S.

Also, tan h = [(tan t)(sin ✓)S]/S since the tangent ratio is opp/adj and S is being substituted for the adjacent side in this ratio.

Therefore, our model could be as follows: tan h = (tan t)(sin ✓).

Because we know the longitude of our specific location, we also know the measure of angle ✓. Let us assume for the sake of easy calculations that we are at a 30^o longitude, and that our style length is 8 inches.

Because we are marking the 11:00 hour line, angle $t = 15^o$. We need to find angle h for several different hours in order to mark the dial plate appropriately. The following is a test calculation:

$$\tan h = \tan 15^o(\sin 30^o)$$
$$\tan h = (.268)(.5)$$
$$\tan h = (.268)(.5)$$
$$h = \tan^{-1}(.134) = 7.63^o$$

Therefore the 11:00 hour line would lie at an angle of 7.63^o to the left of the meridian mark. Also, since the hour marks are symmetrical with respect to the noon mark, it is an angle of 7.63^o to the right of noon for 1:00. We could continue in this fashion to mark the rest of the dial plate for each hour of daylight.

Other modeling activities would be used as primers to get the students to the point where they could successfully manipulate the web experiment in such a way as to define an answer. Because students and teachers both like self contained educational packages, much of what is provided on the web page can really help smooth out problems that might arise during the course of the activities. Other hints that create successful web experiments are as follows:

1. Help students define and modeling heuristic similar to that found on page 2 of this paper, and have this listed on the page as they progress through the activity.
2. Create an on-line hint button that directs students when they are off track. This may even be a step by step derivation of a sample approach to the problem.
3. Use data tables on the web page where students can enter the data from their experiments.
4. Provide a virtual notepad where their models can be entered.
5. Have specific objectives that the activity will highlight, but don't be afraid to deviate. Modeling activities sometimes lead us in directions we may not think of, but turn out to be valid solutions.

As in any educational activity, good planning and lesson design are key to successful implementation. It is important to remember that for every minute of planning on the front end, we save ourselves headaches during the activities themselves.

Conclusion:

Of course the models being presented here take some time to develop and even more time for the students to research. Workload is perhaps the biggest obstacle when it comes to developing on-line math modeling activities. In addition, students and teachers both need to reconceptualize effective mathematics instruction in order to really embrace a modeling approach. Those students who are comfortable in an environment where a math problem can be done in a matter of a few seconds will need to embrace a more realistic view of "scientific" mathematics. In some cases, the students may not come up with the models that we have intended even after hours of work; however, through careful investigation, and with some guidance, students can learn many things that we haven't even thought about. Yet, it is important to remember that many of the greatest inventions of our times have been accidents. Mathematical modeling though provoking problems appears to be a great way to pave the road to accidental learning, and history has taught us that accidents can sometimes be good!